Fabulous Fibonacci
and his nifty numbers

Overview
This series of lessons was designed to meet the needs of gifted children for extension beyond the standard curriculum with the greatest ease of use for the educator. The lessons may be given to the students for individual self-guided work, or they may be taught in a classroom or a home-school setting. This particular lesson plan is primarily effective in a classroom setting. Assessment strategies and rubrics are included. The lessons were developed by Lisa Van Gemert, M.Ed.T., the Mensa Foundation's Gifted Children Specialist.

Introduction

Fibonacci numbers are an interesting mathematical idea. Although not normally taught in the school curriculum, particularly in lower grades, the prevalence of their appearance in nature and the ease of understanding them makes them an excellent principle for elementary-age children to study.

Learning Objectives
After completing the lessons in this unit, students will be able to:
- Explain Fibonacci numbers and their origin.
- Identify Fibonacci numbers in nature and art.
- Generate the next numbers in the Fibonacci sequence.
- Create an original number sequence.
- Create a Fibonacci rectangle and spiral.
- Write an acrostic Fibonacci poem.

Preparation
- Read through the lessons carefully.
- Gather materials.
- Print lessons with a color printer.
Lesson 1: Introduction to Fibonacci numbers

Have you ever pulled the petals off of a daisy? If you look closely at the center of a daisy, you will find that the yellow center is not solid. It is made up of sets of spirals that go out from the center. It’s not just daisies! Nature is all about math.

Look at the bottom of a pinecone. It has those same kinds of spirals. They don’t go around and around in a circle – they go out like fireworks. Look at the pictures below to see what that looks like. How many spirals go in the clockwise direction (green lines)? How many spirals go in a counter-clockwise direction (yellow lines)? Isn’t that strange? Wouldn’t you expect that they would be the same?

To understand the spirals in pinecones, pineapples, daisies and lots of other things in nature, we have to meet a mathematician named Leonardo de Pisa. Most people call him Fibonacci (pronounced fib-o-nawch-ee). About 800 years ago, he wrote a book in which he included a math problem that went like this:

“A certain man put a pair of rabbits in a place surrounded by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair from which the second month on becomes productive?”

(Liber abbaci, pp. 283-284)

(Isn’t it weird that they had word problems 800 years ago?) Fibonacci’s work on this problem led him to this sequence of numbers:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 …

Can you figure out what the next number in the sequence will be?

We call this the Fibonacci sequence, and the numbers are called Fibonacci numbers. To get the next number in the sequence, you add the previous two numbers together. Now go back and look at those pinecone spirals. What do you notice about the number of spirals in each direction, now that you know about Fibonacci numbers?

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
Lesson 2: Finding Fibonacci

Now that you know what Fibonacci numbers are, you’re ready to go on a Fibonacci hunt.

**Flowers**

Look at this flower (it’s a Blizard Bay daylily, image courtesy of Barossa Daylilies). How many petals does it have? Do you see six? Are you thinking that six is not a Fibonacci number? Look more closely. Do you see that there are really two sets of three petals? The outside petals with the straight edges are not really petals. They’re called sepals. So how many actual petals are there?

Not all flowers have three petals. Some have lots more. Some are hard to count because there are so many petals or they are in rows. And there are a few flowers that don’t have Fibonacci numbers for petals. See if you can find a drawing of a daisy. Are the petals a Fibonacci number?

**Seed heads**

The seed heads of flowers are in Fibonacci sequence spirals like you saw in the pinecones. Look at the seed head of this sunflower. Do you see how packed in the seeds are? But none of them gets smashed. The spiral pattern lets the flower fit the most seed heads in the least space.

How do you think you could use this information in your life? Do you think it would work to try to pack a suitcase like this? Why or why not?

**Food**

Look at this picture of a purple cauliflower. If you look carefully, you can see the center of it where the little flowers (florets) begin.

Can you see the spiraling?
Nature is full of Fibonacci numbers and the Fibonacci sequence. Look at this Forestry Service poster of wildflowers. How many examples of Fibonacci numbers can you find in the poster? (Don’t worry that the words are fuzzy; just look at the pictures of the flowers.)
Lesson 3: Working with Fibonacci

Materials
- Large piece of paper
- Graph paper (If you need graph paper, print some at enchantedlearning.com/math/graphs/graphpaper)
- Ruler
- Compass
- Colored pencils

Fibonacci Rectangles
Let’s draw some rectangles using Fibonacci numbers. This will take us to an amazing place. Complete the Fibonacci sequence below (try to do it without help!):

0, 1, 1, 2, 3, 5, __, __, __, __,

Great! You’re ready to start. First, we’re going to do an easy one using graph paper.

1. Start by coloring in one square. This is a 1 x 1 square because it is one length on each side. Now add another 1 x 1 square next to it using a different colored pencil, so it looks like this:

2. Now add a 2 x 2 square so it looks like this:

3. Now add a 3 x 3 square:

4. Now add a 5 x 5 square. Can you do it?

5. Now look at the Fibonacci sequence you wrote down at the top of the page. What size square should you add next? Do it!
The interesting thing about making rectangles like this is that the ratio (the number that shows how the sides relate to each other) stays the same, no matter how big the rectangle gets. This ratio gives us rectangles we call the “Golden Rectangle” because they are said to be the most beautiful rectangles to look at. The ratio is called the Golden Ratio. You can find it by dividing the long side by the short side. So if you have a rectangle that is 3 x 5, you would divide 5 by 3. This will give us a number right around 1.61. (Learn more about the Golden Ratio at http://encyclopedia.kids.net.au/page/go/Golden_ratio)

Can you find some Golden Rectangles around you? How about this piece of paper?

Now you are going to create a Golden Rectangle on your own (on your large piece of paper, not the graph paper) and see something really amazing!

1. First, draw two squares side by side. Use your ruler to make them 0.5 cm square.

2. Now make a 2 x 2 square on top of the first square. So if the first square was 0.5 cm, the 2 x 2 square would be 1 cm square, right?

3. Continue this pattern, making each square the next size in the Fibonacci sequence. So after the 2 x 2 square, you would make a 3 x 3 square (1.5 cm x 1.5 cm), then a 5 x 5 (2.5 cm x 2.5 cm), and so on. This is just like what you did with the graph paper, only using a ruler. Keep going until you have made a square that is 21 x 21 (10.5 cm x 10.5 cm).

4. Each square will have an edge that is the sum of the two squares before it, just like in the Fibonacci sequence.

5. See the diagram below to find out how it should look. The last one is not done yet. Can you see where it would go?
Now we’re going to get really tricky. Are you with me? Get your compass.

Now, with your compass, make an arc in the squares with a radius the size of the edge of the square. Don’t get nervous about these big words; they just mean that the arc will be one-quarter of a circle. Look at the picture below to see what I mean. The arcs in the first squares will be really, really tiny. But look how they grow!

Look at this picture of a nautilus shell. What do you notice?

_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
_________________________
Art Connection
Now look at this painting by Mondrian:

![Composition with Gray and Light Brown by Piet Mondrian](image)

What connections do you find between what you drew and what Mondrian painted? Do all of his rectangles look like Fibonacci-based rectangles to you?

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
Lesson 4: Play with Fibonacci

Threebinacci
Create a sequence of numbers that gets the next number in the sequence by adding the past three numbers.

0, 1, 2, 3, 6, 11, __, __, __, __, __, __, __, __

Now, create your own sequence. How are you going to get the next number in the sequence? Write out your sequence, and then write down the rule for obtaining the next number:

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

Fibonacci Nim
- Number of players: 2
- What you need: Pile of counters (toothpicks, pennies, straws, etc.)
- To play: the first player removes any number (at least one but not all) of the counters from the pile. The second player does the same thing. Each person takes as many as he/she wants, but the player must take at least one counter and no more than two times the number the other player took. The winner is the player who takes the last counter.

If you don’t have anyone to play with you, play by yourself online at nku.edu/~longa/classes/2002fall/mat115/days/day09/newfib.html.
Lesson 5: Write with Fibonacci

For this last lesson, you will create an acrostic poem using the name Fibonacci. In an acrostic poem, the letters in the word form the first letter of each line of the poem. A line may be just one word. Fibonacci has nine letters, so the poem will have nine lines. Look at this example first, using the word “math:"

- Multiplication
- Adding and subtracting
- Teacher helping me understand
- Heaping numbers on top of numbers

Review what you did with Fibonacci numbers and the Fibonacci sequence. Brainstorm a list of words here:

<table>
<thead>
<tr>
<th>Words that go with Fibonacci</th>
<th>Pictures that go with Fibonacci</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fibonacci Acrostic Poem

F

I

B

O

N

A

C

C

I
Extension

- **Additional resources:** library.thinkquest.org/27890/mainIndex.html
- **Fibonacci biography:** traderslog.com/biography-and-fibonacci.htm
- **Printable poster of Fibonacci numbers in nature:** ams.org/ams/ams-printable-posters.html
- **Learn more:** mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html

Images
- Pinecone pictures courtesy of John R. Simmons, University of Georgia Department of Mathematics
- Plant images courtesy of the United States Department of Agriculture
- Fibonacci rectangle construction image courtesy of the University of Minnesota Geometry Center
- Daylily image courtesy of Barossa Daylilies, users.on.net/~daylilies
## Assessment

<table>
<thead>
<tr>
<th>STUDENT CAN:</th>
<th>MASTERY</th>
<th>ADVANCED SKILL</th>
<th>MODERATE SKILL</th>
<th>EMERGING SKILL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate the Fibonacci sequence of numbers without assistance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Create a sequence of numbers with a clear organization pattern</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize Fibonacci numbers in nature and art</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw a Golden Rectangle using squares on graph paper and the Fibonacci sequence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw a Golden Rectangle using a ruler on blank paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>